

MATHEMATICAL MODELLING OF MAIZE (ZEA MAYS, L.) YIELD DUE TO SOIL WATER REGIME

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Abstract

Majerčák, J., Novák, V., Vidovič, J. Matematické modelovanie tvorby úrody kukurice (*Zea mays, l.*) v závislosti na vodnom režime pôdy

Modely pre simuláciu vodného režimu pôdy GLOBAL a pre simuláciu tvorby úrody kukurice (*Zea Mays, L.*) CORNY boli modifikované a použité simultánne ako model CORNWAY na odhad relatívnej úrody. Prvý model (GLOBAL) je matematický deterministický model simulujúci pohyb vody v pôdnom profile počas vegetačného obdobia. Model CORNY je poloempirický model pre výpočet priebežnej redukcie potenciálnej úrody kukurice v dôsledku deficitu vody v pôde počas vegetačného obdobia, ako aj oneskoreného siatia. Za najdôležitejší faktor pri kvantifikácii tejto redukcie sa považuje zasobovanie porastu vodou. Použitá verzia submodelu CORNY je orientovaná na úlohu vody v procese redukcii úrody. Predpokladá sa, že iné faktory nemajú na tento proces vplyv.

Submodels for simulation of soil water regime GLOBAL as well as CORNY to simulate grain yield of maize (*Zea Mays, L.*) can be modified and then used simultaneously as model CORNWAY to estimate the relative plant yields. The first model (GLOBAL) is a mathematical deterministic model which simulates water transport in a soil profile during the vegetation period of crops. Model CORNY is the semiempirical model for calculation of continuous reduction of potential yield of maize crop due to soil water deficit during the vegetation period as well as to planting date delay. The most important factor among others is the water supply of canopy. Version of CORNY submodel as used here is focused on the role of water in yield reduction. It is supposed that the another factors did not limit this process.

Introduction

The effects of water regime of soils on corn yields.

In general the purpose of agricultural water management is to increase the efficiency and reliability of plant production. The goal is consistent with high yields at minimum investment of water, energy and other resources. That means, plants should be avoided of dry and wet stresses, i.e. soil water potential is kept within certain limits out of anaerobic and dry conditions respectively. Drought, and lack of soil aeration can cause decrease of plant production. Mainly in humid regions, artificial drainage systems are installed to satisfy two specific requirements: (1) To insure trafficable conditions for seedbed preparation, planting, harvesting and other field operations. (2) To remove excessive soil water from the root zone during

high rainfall periods so that crop yields are not reduced by oxygen deficiencies or other stresses caused by wet soils. The stress degree also varies from season to season, depending upon weather conditions. To avoid drought situation, it is necessary to irrigate the soil, according to plant demand. The degree of crop stress that occurs due to the deficient and/or excess soil water depends upon the plant growth during which stress occurs [19].

Used models

The mathematical simulation model GLOBAL was developed at the Institute of Hydrology SAS by Majerčák and Novák [7]. It is used as a tool enabling to study the water transport in soil under isothermal conditions. Model GLOBAL is capable to calculate the distribution of water and soil water potentials with respect to time, depending on the initial and boundary conditions. It is assumed that water transport takes place predominantly in vertical direction, horizontal water flows are not considered. It is also assumed that the influence of ionic concentration distribution of dissolved substances upon the water transport is negligible.

Basis of the model is a numerical solution of nonlinear partial differential equation for transport in vertical direction. When solving this equation, an influence of hysteresis of retention curve is considered. Numerical solution in the GLOBAL model is based on the use of Galerkin's modification of the finite element method. A modified version of the numerical solution of Richards equation is used, as it is given by Genuchten [14].

The boundary condition on the soil surface is given by the mean rate of precipitation reduced by interception of canopy, and by intensity of evaporation. In the days without any precipitation, the rate of water transport from soil surface to the atmosphere equals to evaporation intensity. Transpiration rate equals to the rate of water uptake by roots. The lower boundary condition is defined by the mean values of soil potential, or by the vertical rates of water flow in the corresponding depth under the soil surface. The groundwater flow is not the subject of simulation in the GLOBAL model, but can be involved as a kind of boundary conditions.

The basic precondition for the success of the simulation model of water transport is the correct determination of evapotranspiration rates and its structure which enters model as boundary condition. In the GLOBAL model the method of evapotranspiration calculation is used, proposed by Novák [8]. A significant advantage of this method is the possibility of determination of velocity coefficient of turbulent transport of water vapour from evaporating surface to atmosphere.

The submodel CORNY is able in cooperation with submodel GLOBAL (model CORNWAY) to quantify index of water stress in maize (*Zea mays* L.) caused by both excessive and deficient water conditions and to weigh those indexes according to the stage of growing season and crop susceptibility at time they occur. Method is added to predict the yield reduction due to planting delay. Submodel CORNY was implemented to the model GLOBAL using its capability of computation of water content distribution in soil profile and of daily values of actual transpiration. Thus, we can obtain the daily values of stress-day index for wet conditions and the ratio of potential to actual transpiration to compute the drought stress.

In general, model CORNWAY composed of submodels CORNY and GLOBAL is able to give information about possible development of maize yield as affected by changing climatological conditions in future.

General crop response model

The general crop response model scheme used may be written as showed Skaggs et al. [11] as,

$$Y_r = Y_{rw} \cdot Y_{rd} \cdot Y_{rp} \quad (1)$$

where Y_r is the relative yield, $Y_r = Y/Y_o$, $Y_{rw} = Y_w/Y_o$, $Y_{rp} = Y_p/Y_o$. Y is the yield for a given year; Y_o is the potential yield that would be obtained in the absence of the soil water stress conditions including no delay in planting date; Y_w is the yield that would be obtained if only wet stress occur; Y_d is the yield that would be obtained if only drought stresses occur; and Y_p is the yield that would be obtained if the reduction is due to a delay in planting date.

Crop stress factors

Stress – day index concept

The above mentioned crop response model is based on the stress–day index (SDI) concept proposed by Hiler [4]. It assumes that the effect on yields due to the stresses caused by excessive or deficient soil water conditions depend on crop growth stage. The SDI is determined by multiplying a crop susceptibility factor and a stress–day factor. Then the product is summed for all stages of growth

$$SDI = \sum_{i=1}^N CS_i \cdot SD_i \quad (2)$$

where SDI is the stress–day index, CS_i is the crop susceptibility factor, SD_i is the stress–day factor and N is the number of growth stages considered.

Stress factor by wet soil conditions.

Objective function for stress factor by wet soil conditions was proposed by Bouwer [1] as

$$SEW_{30} = \int_0^T f(x) dt \quad (3)$$

t – time during the growing season [days],

T – length of the growing season [days],

x – depth of the water table from the surface [cm].

Function $f(x)$ is defined as $f(x) = 30 - x$ for $x < 30$ and $f(x) = 0$ for $x > 30$.

As suggested by Ravelo [9], crop response by wet soil conditions is related to a stress-day index for wet soil conditions - **SDIW** Daily **SEW**_{30j} values were taken as the stress-day factor and the **SDIW** was calculated as

$$SDIW = \sum_{j=1}^N CS_{wj} \cdot SEW_{30j} \quad (4)$$

where CS_{wj} is the daily crop susceptibility and N is the number of days in the growing season. Crop susceptibility factors for three growth stages of corn for wide scale of climatic conditions and corn varieties are well known. They are presented in Table 1 presented by Skaggs et al. [11]

TABLE 1.

Crop (corn) susceptibility factors CS_w for excessive soil water conditions

Growth stage	Days after planting	Crop susceptibility factor, CS_w
I	0 - 42	0.51
II	43 - 80	0.33
III	81-120	0.02

Stress factor by deficient soil water conditions.

The stress-day index concept was again used to predict the effect of deficient soil water conditions on crop response. The methods used for this component of the model were proposed by Sudar et al. [13]. The stress-day factor (Sudar's water stress index) was taken as $1 - AT/PT$, where AT is the daily actual transpiration total and PT is the daily potential transpiration total. The stress-day index for drought stress, **SDID** (stress day index for dry soil conditions) may be then calculated by the equation

$$SDID = \sum_{j=1}^N CS_{dj} (1 - AT_j/PT_j) \quad (5)$$

where CS_{dj} is the daily crop susceptibility factor for drought stresses and the subscript j indicates the day. N is the number of days in the growing season.

Sudar et al. [13] developed crop susceptibility factors for drought stress using literature data from several sources. Based on his results, the following equations were used to determine the crop susceptibility factor for drought stress in corn, $CS_{dj} = 0$ for $j \leq 26$ (6)

$$\text{and } CS_{dj} = -1.17 + 0.058j - 0.0005j^2 \quad \text{for } j > 26 \quad (7)$$

Relative yield in case of excessive soil water conditions

A relationship between relative yield of corn and **SDIW** (stress day index for wet soil conditions) was developed on the base of field experiments realised by Schwab et al. [12], Ritter and Beer [10], Chaudhary et al. [2], Joshi and Dastane [5]. Complete analysis of all these data was presented by Hardjoamidjojo [3]. The relationships used by above mentioned author for corn are as follows

$$Y_{rw} = 1.0 \quad \text{for } SDIW < 8.0 \quad (8)$$

$$Y_{rw} = 1.03 - 0.0042 SDIW \quad \text{for } 8.0 \leq SDIW \leq 245 \quad (9)$$

$$Y_{rw} = 1.0 \quad \text{for } SDIW > 245 \quad (10)$$

In general, relationships like (8), (9), (10) can be estimated even for different plants, but further field data are needed.

Relative yield in case of deficient soil water conditions

The relationship between relative yield and **SDID** as calculated from Eq. (5) is expressed by Sudar et al. [13] as the following linear function of **SDID**

$$Y_{rd} = 1.0 - 0.112 SDID \quad (11)$$

Effect of planting date delay on yields of corn

Corn yields are significantly reduced even if the planting date is delayed beyond an optimum period. The results of field tests were presented by Krenzer and Fike [6]. Estimating the effect of delay in planting on yields it was assumed that the observed effects are due to such factors as temperature and day length and not to deficient or excessive soil water conditions, which are considered in the other components of Eq. (1). In model CORNWAY are used the following relationships to estimate the effect of planting date delay on relative yield

$$Y_{rp} = 1.0 - 0.0087 PD \quad \text{for } PD < 42 \quad (12)$$

$$Y_{rp} = 1.33 - 0.0166 PD \quad \text{for } 42 < PD \leq 80 \quad (13)$$

$$Y_{rp} = 0 \quad \text{for } PD > 80 \quad (14)$$

where **PD** is the plant date delay [days].

Submodel CORNY

Submodel CORNY is based on the theory, presented above. It was implemented to the model of soil water regime GLOBAL using its capability of precise computation of water-content distribution in soil profile and of daily values of actual transpiration. Thus, we can obtain the daily values of SEW_{30} to compute the stress-day index for wet conditions and the ratio AT/PT to compute the drought stress. The value of planting date delay is entered to calculate the reduction of yield due to this stress factor. In the output of model CORNY are values of relative yield (given by Eq. (1)) and its components, given by equations (2) – (14), as they can be expected at given day.

Results

To validate this model potential yields were simulated during ontogenesis based on field data measured in 1981 for conditions of south-western Slovakia (location Trnava, 48°23' N. lat., 17°35' E. long., 146 m. a.s.l.). Soil was silty chernozem on loess, groundwater table did not influence soil-root zone. The value of simulated relative yield of maize (i.e. ratio of real to potential yield) was 0.29 in physiological ripeness. To verify the model, potential grain dry matter was calculated by two independent methods. The first one was a conventional method based on the effectiveness of the incoming photosynthetic active radiation in grain dry matter yield formation. The second was our modification (Vidovič) [16] of „Wageningen“ method based on the de Wit [17] conception. The values of potential grain dry matter yield calculated by the first, respectively by the second method were 19.49 t.ha⁻¹, and 19.00 t.ha⁻¹, values of the real and calculated potential grain dry matter yields ratios were 5.60/19.49 = 0.287 and 5.60/19.00 = 0.295, respectively. The value of simulated relative yield for the year 1981 was 0.290.

For the year 1982 the value of simulated relative yield was 0.24 and the ratios of real and calculated potential grain dry matter yields ratios were 0.42 and 0.35, respectively.

Conclusions

Model CORNY for simulation of the effect of soil water regime on yield grain of maize (*Zea Mays* L.) was developed. The model is a combination of two models – GLOBAL and CORNY. The first model (GLOBAL) is a mathematical deterministic model which simulates water transport in a soil profile during the vegetation period of crops. Model CORNY is the semiempirical model for calculation of continuous reduction of potential yield of maize crop due to soil water deficit during the vegetation period. The most important factor among others is the water supply of canopy. Version of CORNY model as used here is focused on the role of water in yield reduction. It is supposed that another factors did not limit this process.

Model GLOBAL is able to predict (and to analyse) water dynamics in layered porous media, composed from five layers with different hydrophysical characteristics. The original method of estimation of evapotranspiration and its structure, soil water extraction by roots and was

applied in GLOBAL. Model CORNY was implemented to model GLOBAL using its capability of precise computation of water content distribution in soil profile and of daily values of actual transpiration.

The results of simulated and independently calculated values of real to potential crop yields ratios (i.e. relative yields) are surprisingly close for 1981 year. For the 1982 year the coincidence of the simulated and determined relative grain dry matter yields was not so close as in 1981. The simulated relative grain dry matter yield was 57 % and 67 % of the determined relative yield by the first and second method, respectively. So we have the possibility to predict, on day-by-day basis, the answer of crop (maize) on the soil water conditions and weather factors.

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